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American and Chinese parental involvement in young children's mathematics learning

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Abstract

This study compared the involvement of American and Chinese mothers in their 5- and 7-year-old children's number learning in their everyday experience and during mother–child interaction on mathematics tasks pertaining to proportional reasoning. Results indicated that Chinese mothers of both the 5- and 7-year-old children were more likely to teach mathematics calculation in their everyday involvement with children's number learning than their American counterparts. No differences were found in maternal instruction between American and Chinese mothers during the mother–child interaction on mathematics tasks. However, maternal instruction was related to Chinese children's learning of proportional reasoning but negligibly related to American children's learning of proportional reasoning.

Keywords: Mathematics; Parental involvement; Culture

1. Introduction

Cross-cultural studies have showed that Chinese elementary school children outperform their American peers in mathematics (Stevenson & Stigler, 1992). Many explanations have been proposed to account for this mathematics performance gap, including differences in number-naming systems, the amount of time that children spend on mathematics homework, the content of textbooks, instruction by teachers, classroom organization, parental expectations of children's mathematics achievement, and parental involvement in children's mathematics learning (Chen & Stevenson, 1989; Chen & Uttal, 1988; Crystal & Stevenson, 1991; Li, 2000; Miller, Smith, Zhu, & Zhang, 1995; Perry, VanderStoep, & Yu, 1993; Stigler & Perry, 1988).

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In terms of parental involvement, one of the explanations for Chinese children's superior mathematics performance is that Chinese parents, as compared with American parents, are more frequently involved in their children's mathematics learning. For instance, research has demonstrated that Chinese mothers of first graders spend, on average, almost one hour per day working with their children on homework while American mothers spend an average of 32 min per day on such homework. Some differences have also been found in paternal involvement: 44% of Chinese fathers of first graders spend more than 15 min per day helping their children with homework while only 21% of American fathers do so (Chen & Uttal, 1988).

Despite these results, some recent studies have showed that the relation between parental involvement in children's homework and children's academic achievement is negligible and, in some cases, negative (Levin et al., 1997; Peng & Wright, 1994; Pezdek, Berry, & Renno, 2002). For example, Balli, Wedman, and Demo (1997) conducted a 3-month intervention study and found that 6th grade children who received more parental help on their mathematics homework benefited only slightly as measured on a mathematics posttest compared to children who received less parental help on their mathematics homework.

These recent studies suggest that parental involvement with children's homework does not necessarily improve children's academic performance, and from a comparative perspective they raise an interesting question: Are Chinese parents involved in their children's mathematics learning outside of school in a different and perhaps more effective way than American parents are? To examine this issue, the present study compares American and Chinese mothers on two specific types of parental involvement in children's mathematics learning: (1) maternal involvement in children's number learning outside of school and (2) the interactive processes of mothers and children as they work together on mathematical tasks in a laboratory setting. The laboratory tasks involved proportional reasoning, a type of mathematics understanding that presents difficulty for many children (Piaget, Grize, Szeminska, & Vinh, 1977).

Our investigation of these two types of parental involvement in children's mathematics learning is motivated by two reasons. First, given that Chinese children outperform their American peers in mathematical achievement as early as first grade (Chen & Uttal, 1988; Stevenson et al., 1985), examination of maternal involvement in children's everyday number learning may reveal whether contributions by Chinese parents to young children's mathematics learning are more effective than contributions by American parents. Second, a common limitation in previous studies is that they largely focused on the frequency or amount of time that parents are involved in children's mathematics homework. Little is known about the specific aspects of mathematics learning that parents in these two communities emphasize during these interactions with their children, whether or not the interactions pertain to homework. Perhaps American and Chinese parents differ in their use of instructional techniques that are considered beneficial for mathematics learning, such as instruction that is contingent on the learner's needs (Vygotsky, 1978). The following literature review discusses these two aspects of parental involvement in children's mathematics learning and our interest in young children's learning of proportional reasoning.

1.1. Children's mathematics learning outside of school

An apparent paradox exists in children's mathematical learning. On the one hand, research has documented that young children have abundant opportunities to participate in mathematical activities outside of school (Guberman, 1999; Nunes & Bryant, 1996) and that children across a wide range of cultures can acquire sophisticated mathematical skills through everyday experience (Nunes, Schlieman, & Carraher, 1993; Saxe, 1991). On the other hand, mathematics is widely believed to be one of the most challenging subjects in formal education. Even by the third grade, children begin to experience great discomfort in mathematics learning (Ginsburg, Bempechat, & Chung, 1992). It appears that transferring mathematics knowledge that is acquired outside of school into mathematics learning in school is difficult for many children. Moreover, this difficulty is evident even beyond the classroom setting. Carraher, Carraher and Schliemann (1985) found that Brazilian children who displayed superior arithmetic ability in either real or simulated goods-selling situations experienced great difficulty when similar mathematics problems were presented to them in school-like form as a written mathematics test.

This paradox raises the question of whether some types of everyday experience may promote children's mathematics learning that can be more readily adapted to the classroom context. Given that Chinese children do better in mathematics than their American peers as early as the first grade (Chen & Uttal, 1988; Stevenson et al., 1985), it may be that Chinese parents teach their preschool children mathematical knowledge in the course of everyday experience in ways that benefit their children's later mathematics learning in the school setting. Perhaps Chinese parents use more effective instructional techniques as they work with their children on mathematics tasks than American parents do. Chinese parents may provide more effective mathematical instruction because Asian parents tend to believe that an early start in education is important for children's success at school (Parmar, Harkness, & Super, 2004). This explanation directs attention to theoretical approaches to cognitive development that emphasize social and cultural contributions to intellectual skills, including mathematics learning.

1.2. Sociocultural approach to children cognitive development

Focusing on parental contributions to children's learning is consistent with sociocultural theory (Rogoff, 2003). According to this view, the development of higher mental functions, such as voluntary attention, memory, and reasoning, occurs through the mastery of material and psychological tools and signs. Social interaction plays a crucial role in this development. For Vygotsky (1978), higher mental functions occur first on the interpersonal level and, only later, are these skills present on the intrapersonal level.

A large body of studies conducted in the past two decades have provided empirical support for the idea that social interaction between adults and children contributes to cognitive development in several areas including attention, memory, problem-solving and planning (Gauvain, 2001). Much of this research involves learning tasks that are not typical in the academic setting, such as puzzle completion (Wertsch, McNamee, McLane, & Budwig, 1980), route planning (Gauvain & Rogoff, 1989), and block design (Portes, 1991). However, some research along these lines has been conducted in the domain of mathematics learning. Saxe, Guberman, and Gearhart (1987) observed 2.5- and 4.5-year-old children as they worked on numerical tasks with their mothers. They found that mothers divided the tasks into parts, which the researchers identified as subgoals, and that mothers adjusted the focus of their assistance over the interaction to subgoals that fit with children's changing ability. Children also adjusted their behaviors in relation to the mothers' efforts and they performed better with maternal help than when they worked alone. In another study, Pratt, Green, and MacVicar (1992) observed parents tutor 5th graders on long-division mathematics homework. They found that children whose mothers used more effective scaffolding techniques, such as contingent tutoring, showed more improvement on a mathematics posttest than children who received less effective maternal help. These studies suggest that parental assistance can help children learn mathematics and that assistance that is responsive to the child's learning needs is especially useful. Implicit in this conception are the ideas that parental instruction can benefit children's learning of mathematics and that a child's current abilities and understanding of mathematics influence the type of instruction that parents provide.

1.3. Children's learning of proportional reasoning

Research indicates that mastery of proportional reasoning and the related topic of fractions presents obstacles to mathematics learning for many children (Behr, Harel, Post, & Lesh, 1993; Sophian, 1995). However, interpretations of these difficulties differ among researchers. Piaget and Inhelder (1975) argued that mastery of proportional reasoning requires understanding the relation of relations or second-order relations, which does not occur until children reach formal operations around 11 years or 12 years of age. For Piaget and Inhelder, to understand 1:2 = 2:4, children need first to understand the individual ratios involved, the first-order relations, e.g., that 1:2 means that there is 1 part of something to every 2 parts of something and, likewise, that 2:4 means that there are 2 parts of something to every 4 parts of something. They also need to understand the second order-relation, that is that the two relations or sets of ratios are equivalent (1:2 = 2:4).

Piaget and Inhelder's account of proportional reasoning has been useful to researchers but many questions remain about the development of this understanding and its consequences for reasoning more generally (Nunes & Bryant, 1996). The main developmental questions pertain to the types of proportional reasoning tasks that children younger than 11 years or 12 years of age understand. For some researchers, children's difficulty in understanding proportional reasoning may be because they have trouble understanding the first order relation (i.e., part–part or part–whole relation) involved. Spinillo and Bryant (1991) found that 6- and 7-year-old children could solve proportion problems based on part–part relations, that is how two parts of a whole relate to each other (e.g., whether the amount of blue relative to white in two different size rectangles is equivalent). They also found that "half" boundary information in a visual presentation was important for children to solve these problems. However, later research by Sophian and Wood (1997) found that children as young as 7 years of age can solve proportion problems based on part–whole relations, that is how much of the whole is constituted by a part, but not based on part–part relations and that 5-year-old children cannot use either of these relations to reason about proportions.

Although these findings have yet to be resolved, they both challenge Piaget and Inhelder's claim that before age 11 or 12 children cannot reason about proportions. However, these results may be due, in part, to the types of tasks that were used (Nunes & Bryant, 1996). In both studies pictures were used to represent proportions. Consequently, the results indicate that young children are able to understand proportions using either part–part or part–whole relations based on visual information. Other types of proportion problems may be more difficult for children, such as those that use numerals or exact amounts and, therefore, require different types of operations to solve. Furthermore, as Nunes and Bryant (1996) point out, a proportion is a precise linear relation across two or more variables and quantification may be an essential component of proportional reasoning. Thus, younger children may have difficulty with proportion problems because they lack the mathematical skills, in particular multiplication, needed to solve these problems correctly (Resnick & Singer, 1993). It may also be that young children have success at proportion-like problems that can be solved using other forms of reasoning, such as knowledge of ratios. In fact, Spinillo and Bryant (1991) speculated that children in their study may have treated the problems as ratio problems and that understanding ratios and not proportion accounted for their success.

One interest of the present study was to explore further children's difficulty in learning proportional reasoning. Specifically, we were interested in examining how conceptual and numerical knowledge contribute to children's learning of proportional reasoning, as well as how assistance from mothers may help children in developing this type of understanding. We explore whether young children's difficulty in understanding ratio-related concepts, such as two times, may also account for why they have difficulty in solving proportion problems correctly.

1.4. Hypotheses

According to previous studies, Chinese children perform better in mathematics than American children and Asian parents believe that an early start in education is important for children's success at school (Parmar et al., 2004). Therefore, we predict: (a) Chinese mothers will be more likely than American mothers to teach their children school-oriented mathematical knowledge, such as mathematics calculation, in their everyday involvement with children's number learning to help prepare children for school; (b) instruction by Chinese mothers will include more contingent instruction than by American mothers during an interaction involving mathematics tasks, which, in turn, will enhance children's performance on the tasks; and (c) maternal instruction that focuses on specifying ratio relations involved in proportional reasoning tasks and the interpretation of ratio concepts will be more important for children's learning of proportional reasoning than instruction focused on calculation.

2. Method

2.1. Participants

A total of 72 mothers and their children from the United States and China participated in the study. The 32 American mothers and children were recruited from community centers in a medium size city in Southern California. Half of the children were 5 years of age ($M = 5.45$; S.D. = 2.8 months) and half were 7 years of age ($M = 7.34$; S.D. = 3.4 months). Ethnicity, measured by a parental questionnaire, indicated that 81% of the American participants were European American, 13% were African American or Latino, and 6% were of other ethnicities. The 40 Chinese mothers and children were recruited from two kindergartens and elementary schools in Beijing. Half of the children were 5 years of age ($M = 5.47$; S.D. = 2.8 months) and half were 7 years of age ($M = 7.35$; S.D. = 3.5 months). For both the American and Chinese samples, 50% of the children were boys and 50% were girls. Most of the American (88%) and Chinese (83%) mothers were college educated.

2.2. Procedures

American mothers and children participated during a laboratory visit. Chinese mothers and children participated at the children's school in an empty classroom. Each laboratory or school session began with an interview with the mother about her involvement in children's everyday number learning, after which the mother was asked to complete a questionnaire regarding family demographic information. While the mother completed the questionnaire, the child participated in a solitary mathematics test, in which the experimenter asked the child three counting and five arithmetic problems using toy twigs on which there were different amounts of red plastic berries. For the counting problems, if the child made a mistake in counting, the child was given additional chances to recount the number.

Following the individual mathematics test, the mother and child were given instructions for the interaction tasks, which were 12 problems involving proportional reasoning. During the instructions, the child was asked to recollect the number of berries on the three toy twigs that he or she counted in the individual mathematics test. The 3 twigs had 5, 10, and 20 berries, respectively. After the child reported the number of berries (if the child failed to recollect the numbers, the experimenter gave the child a reminder), the experimenter introduced three different sized troll dolls, identified as Baby, Mommy, and Daddy according to their size. The mother and child were then told that the three dolls ate different amounts of food and a plate was placed in front of each of the dolls. Then the experimenter placed the 3 twigs with 5, 10 and 20 berries on the plates, with one twig on each plate, in front of the Baby, Mommy and Daddy trolls, respectively. The ratio for the amount of food eaten by the three trolls (1:2:4) was not stated explicitly. Following this demonstration, the mother and child were asked to work together to distribute other food to the three dolls. The mother was told that she could talk with her child about the problems in whatever way she thought was best and that there is not any single right or wrong way to do them.

The mother's interview, child's individual mathematics test, and mother–child interaction were audiotaped. To ensure that the experimental procedures were the same across the two cultural groups, both Chinese and American experimenters conducted a pilot test with a separate sample prior to the formal data collection. All research materials, including the interview questions, parent questionnaire, individual mathematics test questions, and the interaction tasks, were designed in English. The Chinese version was developed from the English version by a native Chinese speaker and a back-to-English translation procedure was used to verify that the Chinese and English versions were identical. However, to accommodate the Chinese language, some wording in the mathematics questions had to be changed, though none of these changes pertained to the mathematical content. The word troll in the English version was translated into *little short person* for the Chinese version. Trolls are familiar characters for American children in legends and stories while *little short person* is widely known among Chinese children from the story *Snow White*. Also, the question about mothers' ethnic background in the parent questionnaire was deleted for the Chinese version.

2.3. Measures

2.3.1. Parent interview and questionnaire

Mothers were interviewed about their involvement in children's number learning at home. The initial interview question asked the mother if she ever uses everyday activities to help her child learn about numbers. If the mother answered yes to this question, she was asked two questions about these activities and then completed the demographic questionnaire, described below; if the answer was no, the experimenter skipped immediately to the demographic questionnaire. If the mother stated that she helped her child learn numbers at home, she was asked how frequently she does this, using a 4-point Likert Scale, with 4 representing *almost everyday*, 3 representing *not that frequently but still repeatedly*, 2 representing *maybe once a week*, and 1 representing *once or twice but it doesn't come up that often*. To examine how mothers instruct their children in number knowledge in the course of everyday experience, the mother was asked to describe the last activity in which she helped her child learn about numbers and when it occurred. Mothers could describe any type of activity, such as an activity explicitly focused on number learning, like working on mathematics homework or workbooks with the child, or it could be embedded in an activity in which carrying out the activity entails using numbers, like counting items in a container or playing a game involving calculation.

The specific number-related activities that mothers reported were identified as either involving calculation or not involving calculation. Both calculation and non-calculation activities are number-related only and do not cover other mathematical topics such as space or shape. Calculation activities included number computation, such as addition, subtraction, multiplication, and division. Non-calculation activities did not involve mathematics computation, such as measured amounts (e.g., “Right now, she is taking antibiotics and we talk about taking one teaspoon, and this is how much a teaspoon is.”), reading numbers (e.g., using flashcards to help child identify numerals), and counting (e.g., “Just looking at that book right there. There were rabbits and just looking at the rabbits, one, two, three, you know.”). Following the interview, mothers completed a questionnaire that asked for demographic information, including the child’s age, gender, grade level, and mother’s ethnic and educational background.

2.3.2. Child individual mathematics test

The child-only mathematics test included three counting questions and five arithmetic questions. The 3 counting questions, which assessed children’s counting ability, asked children to count the number of berries on 3 red plastic twigs (each twig was approximately 12 cm in length), containing 5, 10, and 20 berries, respectively.

Of the five arithmetic questions, two were presented in a story about a picnic as follows: (a) three trolls are going on a picnic and each of them needs two goodies. How many goodies will there be in their basket? (b) Two trolls are going on a picnic and they are so hungry that each wants four goodies. How many goodies will there be in their basket? To answer these questions correctly, children could use either addition or multiplication.

The other three arithmetic questions used a blue ribbon (about 90 cm) as a prop. The experimenter took out the ribbon, put her fingers in the midpoint of the ribbon like a scissors and asked the child: See this ribbon? How many pieces would there be if I cut it here? Then, the experimenter folded the ribbon at the midpoint, put her fingers on the midpoint of the folded ribbon and asked: How many pieces would there be if it was cut here? Finally, the ribbon was folded at the midpoint one more time and the same question was asked. The three ribbon questions were designed to assess children’s ability to multiply by two or doubling. To answer the ribbon questions correctly, children need to understand that cutting the ribbon in the midpoint means that the pieces of the ribbon will be doubled.

2.3.3. Interaction tasks

The interaction tasks included 12 mathematical problems that involved distributing different amounts of food to the Baby, Mommy and Daddy trolls. To solve these problems, it is necessary to use the relative sizes of the trolls as a basis for determining the exact share of the food that each troll should receive in each problem given the initial ratio presented in the instructions using the three twigs with berries (5:10:20). Food items were represented by colored stickers with small animal pictures stamped on them, such as frogs, mice and turtles. For 3 of the 12 problems, the mother and child were given the total number of food (7, 14, and 21 stickers) and asked to distribute the food to the three trolls. These problems are referred to as *Total Task Problems*. In the remaining nine problems, the mother and child were told that one troll had eaten a certain amount of food and then they were asked to distribute the remainder of the food to the other two trolls. These problems are referred to as *Anchor Task Problems* because the number of berries that was already eaten by one troll was set or anchored in advance. Half the sample for each age group began with the Total Task Problems and the other half began with the Anchor Task Problems. The three different Total Task Problems and nine different Anchor Task Problems were randomly ordered across the sample. The 12 problems are listed in Appendix A.

2.3.4. Coding and reliabilities

The audiotapes were transcribed by two graduate students and one undergraduate research assistant. All transcriptions were made by native speakers. Information for the individual mathematics test and interview was scored directly from the transcripts. Each maternal statement made during the parent–child interaction that pertained to mathematical content and related to ratios or proportions was coded from the transcripts. Three types of maternal statements were identified: concept-focused instruction, more or less instruction, and calculation-focused instruction.

2.3.5. Concept-focused instruction

These statements pertain to mathematical relations, such as correctly specifying the ratio relations of the problem, e.g., "Mommy wants to eat two times the amount as baby"; identifying the problem as a ratio or proportion problem, e.g., "This is a ratio problem. See the pattern"; teaching the child ratio-related concepts, e.g., "Twice as many means what? Listen, however many the baby has, the mother has that number plus that number again"; and probing of ratio relations or proportions by asking the child why he or she gave a particular amount of food to the trolls, e.g., "Why did you give three frogs to the Baby troll?".

2.3.6. More or less instruction

These statements identify the qualitative difference in the amount of food that the trolls eat without specifying the precise numerical relation, e.g., "Daddy troll eats more than Mommy troll".

2.3.7. Calculation-focused instruction

These statements pertain to counting and mathematical operations used in determining the amount of items eaten by the trolls, that is, the mother counts or prompts the child to count the amount of food eaten by the trolls, e.g., "1, 2, 3, 4, 5, . . ." or "Why don't you count?"; the mother uses or asks the child to use addition or subtraction, e.g., "What is 3 plus 3?"; the mother uses or asks the child to use multiplicative or division operations, e.g., "What is 3 times 2?"; or the mother teaches the child a mathematics calculation, e.g., "Two times 3 is equal to 3 plus 3."

Fourteen (19%) interaction transcripts, of which eight were Chinese and six were English, were randomly drawn from the total of 72 transcripts and coded by two independent coders, fluent in the appropriate language, to calculate interrater reliability. The Kappa reliability coefficients for each code were as follows: interpretation of ratio relations (.90), identification of problem as a ratio problem (.82), ratio instruction (.88), probing of ratio relations (.90), more or less instruction (.87), counting (.85), addition or subtraction (.82), multiplication and division (.87), and calculation instruction (.85).

2.3.8. Child individual mathematics test performance

Both 5- and 7-year-old American and Chinese children approached or were at ceiling on the counting problems. Therefore, the number of answers to the five arithmetic problems that the child calculated correctly was used to index the child's performance on the individual mathematics test.

2.3.9. Child interaction task performance

Child performance during the interaction tasks was calculated as the number of answers to the 12 interaction problems that the child solved correctly on his or her own without any mathematics help from mother. The Kappa interrater reliability of this variable was .93.

3. Results

Results are presented in relation to the following cross-cultural comparisons: (a) maternal involvement in children's number learning at home; (b) children's individual mathematics test and interaction task performances; (c) maternal instruction during the interaction tasks and its relation to children's individual mathematics test performance; (d) the prediction of child age, the frequency of maternal involvement in children's number leaning, and children's individual mathematics test performance to children's interaction task performance; and (e) the prediction of child age and maternal instruction during the interaction tasks to children's interaction task performance. Preliminary analyses indicated that there were no differences due to child gender or order of presentation of the Anchor and Total Task problems. Therefore, all subsequent analyses collapsed the data on these dimensions.

3.1. Maternal involvement in children's number learning at home

There was no difference in the responses of Chinese and American mothers of 5-year-old children to the interview question as to whether the mothers help their children with number learning. All of the mothers of 5-year-old children answered yes to this question. However, a difference in the responses was found between American and Chinese mothers of 7-year-old children. Thirteen of the 16 American mothers of 7-year-old children answered yes to the question, whereas all 20 Chinese mothers of 7-year-old children answered yes, χ^2 (1, $N = 36$) = 4.09, effect size $r = .34$, $p < .05$.

A 2 × 2 (age × country) ANOVA was used to examine responses by American and Chinese mothers to the second interview question of how frequently they are involved in their 5- and 7-year-old children's number learning. Frequency scores were based on a 4-point Likert scale (4 = almost everyday). No significant main effects were found for child age, $F(1, 68) = .03$, *ns*, or country, $F(1, 68) = .18$, *ns*. However, a significant interaction was found between child age and country, $F(1, 68) = 4.38$, $p = .04$, $\eta^2 = .06$. To explore the nature of the interaction, the mean frequency for each group was examined (see Fig. 1). Post-hoc analysis showed that the difference in the frequency scores between American and Chinese mothers was significant for the 7-year-old group ($M = 2.44$, $M = 3.20$; $t(34) = 2.11$, effect size $r = .34$, $p = .04$, but it was not significant for the 5-year-old group ($M = 3.06$, $M = 2.75$; $t(34) = .85$, $p = .40$).

The types of activities that Chinese and American mothers reported that they used to help their children learn about numbers were compared using chi-square analysis (see Table 1). Significant differences were found in the activities used by American and Chinese mothers in both the 5- and 7-year-old groups, χ^2 (1, $N = 36$) = 11.90, effect size $r = .57$, $p = .001$; χ^2 (1, $N = 34$) = 23.68, effect size $r = .83$, $p < .001$, respectively. Chinese mothers of both 5- and 7-year-old children were more likely to teach their children mathematics calculation ($n = 28$) than American mothers ($n = 5$) were. In addition, five Chinese mothers reported teaching their children counting, two mothers reported teaching their children about mathematical units, and one mother each reported teaching children about fractions, word problems, mathematical symbols, and a mathematical thinking method (not further specified). Other than the five American mothers who reported teaching their children about calculation, the remaining mothers in this group reported teaching both 5- and 7-year-old children about counting ($n = 13$), measurement ($n = 2$), computer games ($n = 2$), and one mother each reported teaching children about fractions, read-

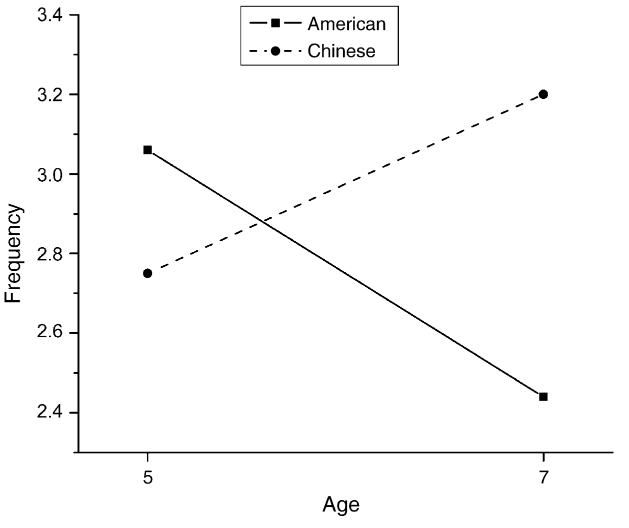

Fig. 1. Mean frequency of maternal involvement in children's everyday mathematics activities as a function of child age and country.

ing numbers, remembering numbers, identifying numbers on flashcards, and concepts of time and money.

3.2. Child individual mathematics test and interaction task performances

The individual mathematics test and interaction task performances of American and Chinese children were compared using independent *t*-tests. Both 5- and 7-year-old Chinese children performed better on the individual mathematics test and interaction tasks than their American peers (see Table 2). One-way ANCOVA was used to examine whether 5- and 7-year-old Chinese children's better individual mathematics test performance contributed to their better interaction task performance as compared with American children. Results showed that 7-year-old Chinese children still did better on the interaction tasks than American children, $F(1, 32) = 4.59$, $\eta^2 = .13$, $p = .04$, when the individual mathematics test performance

Table 1
Reported calculation activities at home of American and Chinese mothers and their 5- and 7-year-old children

Country	Calculation	
	Yes	No
5-year-old group		
American	2	14
Chinese	14	6
7-year-old group		
American	3	13
Chinese[a]	18	0

[a] One Chinese mother's data were missing and one Chinese mother's audiotape about the reported activity was not clear.

Table 2
Means (and standard deviations) and mean comparisons of children's individual mathematics test and interaction task performances

Performance	5-year-old group			7-year-old group		
	American *M* (S.D.)	Chinese *M* (S.D.)	*t*	American *M* (S.D.)	Chinese *M* (S.D.)	*t*
Mathematics test	1.56 (.96)	3.89 (.46)	8.86***	3.12 (1.31)	4.05 (.85)	2.44*
Interaction task	2.06 (2.26)	3.85 (2.87)	2.03*	3.63 (2.19)	6.05 (2.96)	2.73**

* $p < .05$.
** $p < .01$.
*** $p < .001$.

was partialled out, but this pattern did not appear for 5-year-old group, $F(1, 32) = .02$, $p = .90$.

3.3. Maternal instruction during the interaction and in relation to children's individual mathematics test performance

Independent *t*-tests were used to examine whether there were mean differences in the three types of maternal instruction – concept-focused instruction, more or less instruction, and calculation-focused instruction – between American and Chinese mothers of both 5- and 7-year-old children during the interaction. No significant differences were found on the three types of instruction (Table 3). It is noteworthy that a marked proportion of American (34.5%) and Chinese (37.5%) mothers did not identify the ratio relations involved in the interactions tasks and did not provide conceptual instruction during the interaction. To explore this pattern in relation to task performance, American mothers who provided concept-focused instruction solved 92% of the interaction problems correctly and American mothers who did not provide such instruction solved 40% of these problems correctly. Chinese mothers who provided concept-focused instruction solved 98% of the interaction problems correctly and Chinese mothers who did not provide such instruction solved 52% of these problems correctly.

Zero-order correlation was used to examine whether children's individual mathematics test performance was correlated with maternal instruction during the interaction tasks. Results showed that both American and Chinese mothers' concept-focused ($r = .03$, $p = .87$; $r = -.16$, $p = .33$,

Table 3
Means for types of instruction provided by American and Chinese mothers

	5-year-old group			7-year-old group		
	American *M* (S.D.)	Chinese *M* (S.D.)	*t*	American *M* (S.D.)	Chinese *M* (S.D.)	*t*
Concept-focused	11.69 (12.89)	13.65 (14.09)	−.43	7.75 (9.71)	9.60 (16.60)	−.42
Calculation-focused	17.18 (13.52)	21.00 (14.31)	−.81	13.56 (16.60)	10.25 (12.20)	.69
More or less	5.81 (5.92)	3.25 (5.92)	1.29	3.31 (5.70)	3.10 (7.57)	.09

Table 4
Summary of hierarchical regression of child age, maternal involvement in children's number learning at home, and children's individual test performance predicting children's interaction task performance

Variable	*B*	S.E. *B*	*β*
American			
Step 1			
Age	.78	.39	.34*
Step 2			
Age	.98	.38	.43*
Frequency of maternal involvement	.64	.29	.36*
Step 3			
Age	.63	.47	.28
Frequency of maternal involvement	−.59	.29	.34*
Children's individual test performance	.43	.33	.25
Chinese			
Step 1			
Age	1.16	.48	.37*
Step 2			
Age	.94	.50	.30
Frequency of maternal involvement	.82	.56	.23
Step 3			
Age	.85	.50	.27
Frequency of maternal involvement	−.91	.56	.26*
Children's individual test performance	.90	.72	.19

Note: For American sample, $R^2 = .12$ for Step 1 ($p = .05$), $\Delta R^2 = .12$ for Step 2 ($p = .04$), $\Delta R^2 = .04$ for Step 3 ($p = .21$); for Chinese sample, $R^2 = .14$ for Step 1($p = .02$); $\Delta R^2 = .05$ for Step 2 ($p = .15$), $\Delta R^2 = .04$ for Step 3 ($p = .22$).
* $p < .05$.

respectively) and calculation-focused instruction ($r = -.03$, $p = .88$; $r = -.24$, $p = .15$, respectively) were negligibly correlated with children's individual mathematics test performance.

3.4. Do child age, maternal involvement in children's number learning at home, and children's individual mathematics test scores predict children's mathematics performance during the interaction?

Results of hierarchical multiple regression analyses for the American and Chinese samples are summarized in Table 4. It was assumed that 7-year-old children would perform better than 5-year-old children, and therefore child age was entered first in the analysis so that the potential age effect could be controlled. For the American sample, child age was a significant predictor of children's mathematics performance during the interaction at the Step 1 ($R^2 = .12$, $p = .05$). The addition at Step 2 of maternal involvement in children's number learning at home increased the explained variance significantly, $\Delta R^2 F(1, 29) = .12$, $p = .04$. However, the addition of children's individual mathematics test performance at Step 3 did not increase the explained variance, $\Delta R^2 F(1, 28) = .04$, $p = .21$. For the Chinese sample, child age was a significant predictor at Step 1 ($R^2 = .14$, $p = .02$). The addition of maternal involvement at Step 2 ($\Delta R^2 F(1, 35) = .05$, $p = .15$) and children's individual mathematics test performance at Step 3 ($\Delta R^2 F(1, 34) = .04$, $p = .22$) did not increase the explained variance significantly.

Table 5
Regression analysis for child age and maternal instruction predicting American and Chinese children's interaction task performance

Instruction variable	*B*	S.E. *B*	β
American			
Step 1			
Age	.78	.39	.34*
Step 2			
Age	.81	.40	.36*
Calculation-focused instruction	.21	.38	.10
Step 3			
Age	.81	.41	.35
Calculation-focused instruction	−.23	.66	−.11
Concept-focused instruction	.42	.52	.25
Chinese			
Step 1			
Age	1.10	.46	.36
Step 2			
Age	.40	.45	.13
Calculation-focused instruction	−1.60	.45	−.53***
Step 3			
Age	.20	.42	.07
Calculation-focused instruction	−2.48	.51	−.81***
Concept-focused instruction	.93	.33	.44***

Note: For American sample, $R^2 = .12$ for Step 1 ($p = .05$); $\Delta R^2 = .01$ for Step 2 ($p = .59$), $\Delta R^2 = .02$ for Step 3 ($p = .43$); for Chinese sample, $R^2 = .13$ for Step 1($p = .02$); $\Delta R^2 = .22$ for Step 2 ($p = .001$), $\Delta R^2 = .12$ for Step 3 ($p = .007$).
* $p < .05$.
*** $p < .001$.

3.5. Do child age and maternal instruction during the interaction predict children's mathematics performance during the interaction?

Hierarchical regression was used to examine child age and the three types of maternal instruction during the interaction tasks (i.e., concept-focused instruction, more or less instruction, and calculation-focused instruction) in relation to American and Chinese children's interaction task performance, respectively. Because the raw data of the three instruction variables were not normally distributed for both the Chinese and American samples, a natural logarithm function was used to transform the raw data to minimize the skewness of the distributions. The variable of more or less instruction was resistant to normal transformation, and therefore, was dropped from the regression analysis. The results for child age and the remaining two instruction variables are summarized in Table 5.

A different prediction pattern was found for the American and Chinese samples. For the American sample, child age was a significant predictor of children's interaction performance at Step 1 ($R^2 = .12$, $p = .05$). Neither the addition of maternal calculation-focused instruction at Step 2 ($\Delta R^2 F(1, 29) = .01$, $p = .59$) or concept-focused instruction at Step 3 ($\Delta R^2 F(1, 28) = .02$, $p = .43$) increased the explained variance. For the Chinese sample, age was a significant predictor of children's interaction task performance at Step 1 ($R^2 = .12$, $p = .02$). Both the addition of maternal calculation-focused instruction at Step 2 ($\Delta R^2 F(1,37) = .22$, $p = .001$) and concept-focused instruction at Step 3 ($\Delta R^2 F(1,36) = .12$, $p = .007$) increased the explained variance significantly.

4. Discussion

The present study compared two types of parental involvement in young children's mathematics learning in an American and a Chinese sample: maternal involvement in children's number learning at home and maternal instruction during mother–child interaction on mathematics tasks pertaining to proportional reasoning.

4.1. Maternal involvement in children's number learning at home

All American and Chinese mothers of 5-year-old children reported that they are involved in their children's number learning at home while Chinese mothers of 7-year-old children reported more of this involvement than their American counterparts. These findings are consistent with the results of previous studies (Stevenson & Stigler, 1992), which indicated that American mothers tend to pull back from children's academic instruction after children begin elementary school while Chinese mothers devote more time and effort to their children's academic learning after children enter school. This difference may also reflect cultural differences in parental expectations regarding children's academic performance. Compared to American parents, Chinese parents have higher expectations of their children's academic achievement (Stevenson, 1993), which may motivate them to be more involved in their children's academic learning and lead them to emphasize school-oriented mathematics skills, particularly calculation, after children enter school. This difference may also be related to cross-cultural differences in young children's counting competence. According to Miller and his colleagues (1995), Chinese children display more counting competence at age 5 than their American peers do. Such competence may make the teaching of calculation skills by Chinese mothers to their children more possible.

For mothers' instruction of number knowledge to their children, our results indicated that Chinese mothers reported teaching their early school-age children more number-related knowledge than American mothers did. However, these results cannot be generalized to other areas of mathematics. It is possible that American mothers concentrate on other mathematics topics in their interactions with their children, such as shape and spatial relations. We found both overlap and differences in the descriptions provided by the American and Chinese mothers of the types of mathematics activities they use at home to help their children learn about numbers. Further studies should be conducted to investigate the nature and the extent of involvement at home of American and Chinese parents in children's number learning as well as other areas of mathematics learning that are important to children's success at school.

4.2. Maternal involvement in number learning at home, maternal instruction during the interaction, and children's performances on the individual mathematics test and interaction tasks

Both 5- and 7-year-old Chinese children performed better on the individual mathematics test and the interaction tasks than their American counterparts. This pattern is consistent with previous studies in which Chinese children did better on mathematics tests than their American peers (Stevenson & Stigler, 1992). Given that Chinese mothers reported being less involved in their 5-

year-old children's number learning than American mothers did, the better individual mathematics test performance by 5-year-old Chinese children may be related to Chinese mothers' emphasis on calculation in their involvement with children's number learning. In other words, the specific mathematics knowledge that parents teach children may play a more important role in promoting the mathematics competence of preschool children than the frequency of parental involvement in children's number learning.

Although the interaction task performance both of 5- and 7-year-old Chinese children was also better than their American peers, when we controlled for children's individual mathematics test performance in the analysis, 7-year-old but not 5-year-old Chinese children did better than their American counterparts. We hypothesized that better performance on the interaction task by Chinese children would be due to mathematical help provided by Chinese mothers during the interaction. Contrary to our hypothesis, there were no differences in the instruction provided by American and Chinese mothers during the interaction tasks. This result also seems inconsistent with the finding that Chinese mothers were more likely to teach their children calculation at home than American mothers were. We suggest that the inconsistency between the two findings might be because parents teach mathematics knowledge to their children based on children's present mathematics competence as revealed in their everyday experience with their parents. In other words, parental instruction is oriented to both parent's perception of their child's knowledge and to task demands. As noted previously, Chinese children display a higher counting competence at a younger age than American children, which may have encouraged Chinese parents to teach at a higher level of mathematics knowledge than American parents. However, the demands of the proportional reasoning tasks used in this study may have led parents in both groups to include calculation and ratio concepts in their instruction. Thus, task demands may have reduced the difference in instruction between American and Chinese mothers.

Results of hierarchical regression analysis revealed different patterns of prediction of children's performance on the interaction tasks in the two cultural groups. For American children, the frequency of maternal involvement in children's number learning at home predicted children's interaction task performance. This result indicates that the frequency of American mothers' instruction of number-related knowledge at home may contribute to children's learning a new mathematics topic, like proportional reasoning in the present study. However, for Chinese children, maternal instruction during the interaction was a significant predictor of children's performance on the interaction tasks. Specifically, both calculation-focused instruction and concept-focused instruction were related to better performance by Chinese children on the proportional reasoning tasks. Discussion of the relations between maternal instruction during the interaction tasks and children's performance on these tasks may help in interpreting these patterns.

4.3. Maternal instruction during the interaction and children's learning of proportional reasoning

Maternal concept-focused instruction and calculation-focused instruction during the interaction tasks had different relations to Chinese and American children's performance on these tasks. For the Chinese sample, maternal calculation-focused instruction had a significant negative relation with Chinese children's interaction task performance. This finding may be related to Chinese children's relatively higher calculation competence, as revealed in the individual mathematics test. When children already possess relatively high calculation competence, maternal calculation instruction may distract children's attention from other aspects of the mathematics tasks. Chinese mothers' concept-focused instruction had a significant positive relation with children interaction task performance after maternal calculation-focused instruction was added into the hierarchical regression equation. This result provided support for our hypothesis that maternal instruction of ratio relations and ratio concepts is important for young children to learn proportional reasoning.

For the American sample, neither maternal calculation-focused instruction nor concept-focused instruction predicted children's interaction task performance. The negligible relations may be related to American children's calculation competence, which was revealed in the individual mathematics test as weaker than that of the Chinese children. Less calculation competence may have limited the American children's ability to solve proportional-reasoning problems successfully. In this sense, these results provide support for Resnick and Singer's (1993) perspective that numerical knowledge plays an important role in solving proportional reasoning problems. In all, the results indicated that both understanding of ratio concepts and basic arithmetic competence are important for young children's learning of proportional reasoning problems and that young children may learn to solve proportional reasoning tasks with help from adults.

These findings also provide support for previous studies that showed that parental involvement in children's mathematics education may have negligible and negative effects on children's number learning (Balli et al., 1997; Pezdek et al., 2002). It is noteworthy that 34.5% of the American mothers and 37.5% of the Chinese mothers in this sample did not mention the ratio relations underlying the interaction tasks to their children. Although we cannot know why these mothers did not mention this information to their children, which would have been useful to learning about the tasks, it may be that some of these mothers did not recognize or understand the ratio relations or proportions. Further examination revealed that mothers in both cultural groups who did not mention ratio relations solved, on average, 46% of the proportion problems whereas mothers who mentioned these relations solved, on average, of 95% of the problems. This pattern suggests that adults may not be as competent as they are often assumed to be in terms of the abilities they need to help their children, even in the early school years, to learn mathematics. The fact that most of the mothers in the study were college educated, even though college education differs across these two cultures, makes this pattern particularly significant. In addition, some aspects of parental mathematics instruction, such as Chinese mothers' calculation-focused instruction, were negatively related to children's learning of proportional reasoning. This suggests that not all parental help or instruction may contribute to children's learning or understanding of mathematics.

One limitation of the present study is that the sample size of American and Chinese participants was not large. In addition, because most of mothers were college educated, we cannot generalize the findings of the present study to mothers in the population who do not receive education beyond high school.

5. Conclusions

Taken together, the results of this study suggest the following conclusions. Parental involvement helps children learn mathematics concepts and how to carry out mathematical operations. However, it is the nature of this involvement and not merely the amount of time that parents are involved in children's number learning that matters. During everyday experience, the specific mathematics knowledge that parents teach their children appears to be more important for young children's number learning than the amount of time that parents are involved in helping their children learn mathematics. In addition, the number knowledge that parents emphasize in their involvement with children at home varies across cultural groups, which, in turn, may have different consequences for children's learning. Specifically, Chinese mothers' emphasis on teaching their young children school-oriented mathematics knowledge, particularly calculation, may help explain why Chinese children perform better in school mathematics relative to American children.

Contrary to expectations, our observations of parent–child interaction on the mathematics tasks revealed no difference in the instructional techniques used by American and Chinese mothers. However, we found that after controlling for the children's individual mathematics test performance, 7-year-old Chinese children still did better on the interaction tasks involving proportional reasoning than their American peers. This finding suggests that Chinese mothers may have presented mathematical knowledge in a more effective way than the American mothers did even though the mathematics knowledge that American and Chinese mothers provided to their children was similar and the majority of mothers in both groups was college educated. However, it is important to note that the college entry requirements in China, in general, are stricter than in the United States. Therefore, the sample of Chinese mothers might have been more homogeneous and better educated than the sample of American mothers, This difference may have enabled Chinese mothers to attune to their children's learning needs better and, thereby, provide more effective instruction.

The findings that mothers' calculation-focused instruction was not particularly useful for children's learning of proportional reasoning in the Chinese sample and the negligible relations between maternal instruction and children's interaction task performance in the American sample suggest that although more experienced partners may play a critical role in supporting cognitive development (Vygotsky, 1978), adult assistance does not always support children's learning. Better understanding of the conditions in which adult assistance supports children's learning is needed.

American and Chinese mothers provided similar amounts of concept-focused instruction. However, Chinese children benefited from this instruction, as measured by their performance on the proportional reasoning tasks during the interaction. This result indicates that maternal instruction of ratio concepts is important for young children's learning of proportional reasoning. Such instruction may help children understand the first order relation – ratio relations – involved in proportional reasoning problems. The differential benefit of this information in the two cultural groups is puzzling, however. The fact that Chinese children had better individual mathematics test performance than American children suggests that Chinese children may have had better numerical knowledge going into the interaction. This knowledge may have helped Chinese children understand maternal concept-focused instruction better than American children, which, in turn, may have contributed to their learning of proportional reasoning. Thus, children's numerical knowledge may also be critical for children's learning of proportional reasoning.

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Appendix A. Interaction problems

The Three Total Task Problems. Task (1): The experimenter puts out a total of seven lizards and says: This time, they have this many lizards and the trolls want to eat all of them. Please, put the right number of lizards on each of the three trolls' plates (Answers: 4 2 1). Task (2): The experimenter puts out a total 14 ladybugs and says: Let's try a new one. They have this many ladybugs. Please put the right number of ladybugs on each of the trolls' plates (Answers: 8 4 2). Task (3): The experimenter puts out a total of 21 mice and says: The trolls are having another meal. Look what they have. Please put the right number of mice on each of the trolls' plates (Answers: 12 6 3).

The Nine Anchor Task Problems. "Now we're going to play another game. Do you remember that this is baby's plate and this is mama's plate and that's daddy's plate?" The experimenter then states that one of the trolls is full (Baby, Mama or Daddy troll). For Task 4, for example, the family is eating lizards and Mama is eating six and she is full and doesn't want to eat any more. "So you need to figure out how many ladybugs are there for Daddy and for baby to eat." The specific items and anchors were as follows. Task (4) Total of 12 lizards, with 6 on Mama's plate (Answers: 12 6 3). Task (5) Total of 14 ladybugs, with 8 on Daddy's plate (Answers: 8 4 2). Task (6) Total of 7 turtles, with 2 on Mama's plate (Answers: 4 2 1). Task (7) Total of 7 turtles, with 4 on Daddy's plate ((Answers: 4 2 1). Task (8) Total of 21 frogs, with 3 on Baby's plate (Answers: 12 6 3). Task (9) Total of 7 lizards, with 1 on Baby's plate (Answers: 4 2 1). Task (10) Total of 14 ladybugs, with 4 on Mama's plate (Answers: 8 4 2). Task (11) Total of 21 frogs, with 12 on Daddy's plate (Answers: 12 6 3). Task (12) Total of 14 mice, with 2 on Baby's plate (Answers: 8 4 2).

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